

# Fast geometric gate operation of superconducting charge qubits in circuit QED

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A scheme for coupling superconducting charge qubits via a one-dimensional superconducting transmission line resonator is proposed. The qubits are working at their optimal points, where they are immune to the charge noise and possess long decoherence time. Analysis on the dynamical time evolution of the interaction is presented, which is shown to be insensitive to the initial state of the resonator field. This scheme enables fast gate operation and is readily scalable to multiqubit scenario.

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Quantum computers have been paid much attention in the past decade and solid state systems are promising candidates for novel scalable quantum information processing [1]. In particularly, the idea of placing superconducting qubits inside a cavity, i.e., the circuit quantum electrodynamics (QED), has been illustrated [2, 3] to have several practical advantages including strong coupling strength, immunity to noises, and suppression of spontaneous emission.

Decoherence always occur during real quantum evolutions and therefore stands in the way of physical implementation of quantum computers. So, how to suppress this infamous decoherence effects is a main task for scalable quantum computation. To fight against cavity decay, in typical circuit [3] and cavity [4] QED systems, convectional wisdom is to resort to the so-called large detuning interaction method. Similarly, in trapped thermal ions system, the famous bichromatic excitation scheme [5] couples ions by virtue excitation of phonon mode, also uses the large detuning interaction. Later investigation shows that the logical operation obtained is of the geometric nature [6] and therefore has high fidelity [7]. Meanwhile, it is shown that by periodically decoupling to the common phonon mode, the large detuning constrain can be removed [6] so that fast gate operation can be achieved [8]. Similar strategy can be adopted in cavity QED system with strong driven atoms [9], superconducting charge qubits in a microwave cavity by introducing ac magnetic flux [10] and superconducting flux qubits inductively coupled to a common resonator [11].

In typical circuit QED system, up to now, theory and experimental explorations are still in the stage of large detuning interaction. Here, we propose to couple superconducting charge qubits via a one-dimensional (1D) superconducting transmission line resonator (cavity). The qubits are capacitively coupled to the 1D

superconducting cavity [2] and work at their optimal points, where they are immune to the charge noise and possess long decoherence time. The gate operation is shown to be insensitive to the initial state of the cavity field, and thus greatly suppress the decoherence effect from the cavity decay. This scheme removes the requirement of large detuning, and thus enables fast gate operation. Finally, the solid-state set-up is readily scalable to multiqubit scenario.

Before proceeding, we would like to explain our proposal in a more physical way. Usually, 2-qubit coupling is demonstrated with large qubit-cavity detuning  $\delta \gg g$ , e.g., in Ref. [2], which makes the coupling quite weak. In this regime, there is no energy exchange between qubits and cavity. The effective coupling of energy conservation transitions can be determined by second-order perturbation theory [6]. Meanwhile, the coupling usually contains cavity-state-dependent energy shift, i.e.,  $a^\dagger a \sigma_j^z$  terms. Outside this regime the internal state is strongly entangled with the cavity state during the gate operation. By adding a driven field to the cavity field, we get effective driven for all qubits, which is similar to scheme of atomic cavity QED with strong driven [12]. This driven field further mix both the cavity and the qubit state. For successful gate operation (evolution independent of the cavity state) we have to ensure that the cavity returns to its initial state at the end of the gate. Fortunately, this can be achieved by appropriately chosen parameters, i.e., to fulfill Eq. (18), where the two mix mechanisms of the cavity field state conceal each other and thus result in cavity state independent evolution of the system.

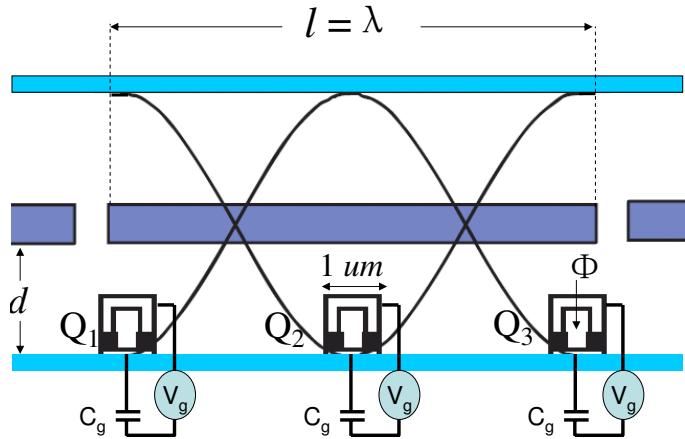


FIG. 1: (Color online) The architecture of 3 superconducting charge qubits capacitively coupled to a 1D cavity, which consists of a full-wave section ( $l = \lambda$ ) of superconducting coplanar waveguide. Qubits are placed between the superconducting lines and is capacitively coupled to the center trace at a maximum of the voltage standing wave (solid cosine curves), and thus yielding maximum coupling. Qubit consists of two small Josephson junctions in a  $1 \mu m$  loop to permit tuning of the effective Josephson energy by an external flux  $\Phi$ . Input and output signals can be coupled to the resonator, not shown here, via the capacitive gaps in the center line.

Fig. (1) shows our proposed setup with 3 qubits. The 1D cavity consists of a full-wave section ( $l = \lambda$ ) of superconducting coplanar waveguide. A distinct advantage of circuit QED approach is that the zero-point energy is distributed over a very small effective volume, which leads to significant rms voltages between the center conductor and the adjacent ground plane at the antinodal positions. At a cavity resonant frequency of 10 GHz and  $d = 10 \mu\text{m}$ ,  $V_{rms} = 2 \mu\text{V}$  corresponding to electric fields  $E_{rms} = 0.2 \text{ V/m}$ , which is several hundreds times larger [2] than that of achieved in 3D cavity with Rydberg atoms.

The superconducting charge qubit considered here consists of a small superconducting box with excess Cooper-pair charges, formed by an symmetric SQUID with the capacitance  $C_J$  and Josephson coupling energy  $E_J$ , pierced by an external magnetic flux  $\Phi$ , permit tuning of the effective Josephson energy. A control gate voltage  $V_g$  is connected to the system via a gate capacitor  $C_g$ . Focus on the charge regime ( $E_J \ll E_c = 2e^2/C_\Sigma$  with  $C_\Sigma = C_g + 2C_J$ ), at temperatures much lower than the charging energy ( $E_c$ ) and restricting the induced charge [ $\bar{n} = C_g V_g / (2e)$ ] to the range of  $\bar{n} \in [0, 1]$ , only a pair of adjacent charge states  $\{|0\rangle, |1\rangle\}$  on the island are relevant. Then, the device is described by [1]

$$H_s = -\frac{\epsilon}{2}\tilde{\sigma}^z - \frac{\Delta}{2}\tilde{\sigma}^x, \quad (1)$$

where  $\epsilon = E_c(1 - 2\bar{n})$ ,  $\Delta = 2E_J \cos(\pi\Phi/\phi_0)$  with  $\phi_0$  being the flux quanta,  $\tilde{\sigma}^x$  and  $\tilde{\sigma}^z$  are the Pauli matrices in the  $\{|0\rangle, |1\rangle\}$  basis. Note that the qubit splitting now can be tunable by the external magnetic flux, which can be used to turn on/off cavity mediated qubits interaction. As this can be tuned individually, the cavity mediated qubits interaction can be implemented on selective qubits.

The qubits are capacitively coupled to the cavity, as shown in Fig. (1). For simplicity, we here assume that the cavity has only a single mode that plays a role. To obtain maximum coupling strength, they are fabricated close to the voltage antinodes of the cavity. As the wave length of the cavity mode ( $\lambda \sim 1 \text{ cm}$ ) is much larger than the linear length of the qubit ( $1 \mu\text{m}$ ), we may treat the qubit-cavity coupling as a constant within the qubit geometry. This coupling is determined by the gate voltage, which contains both the dc contribution and a quantum part. If the qubit is placed in the center of the resonator, as shown in Fig. (1), the latter part contribution is given by  $V_{rms}^0(a + a^\dagger)$  with  $V_{rms}^0$  being the rms voltage between the center conductor and the ground plane. Then, the Hamiltonian describes this setup takes the form of [2]

$$H_c = \omega_r a^\dagger a - \sum_{j=1}^N \left[ \frac{\epsilon}{2}\tilde{\sigma}_j^z + \frac{\Delta}{2}\tilde{\sigma}_j^x + g_j(a + a^\dagger)(1 - 2\bar{n} - \tilde{\sigma}_j^z) \right], \quad (2)$$

where we have assume  $\hbar = 1$ ,  $\omega_r$ ,  $a$  and  $a^\dagger$  is the frequency, annihilation and creation operator of the cavity field, respectively; the coupling strength of  $j$ th qubit to the cavity is  $g_j = eC_{g,j}V_{rms}^0/C_{\Sigma,j} \in [5.8, 100]$  MHz [2]. Rotate to the qubit eigen basis of

$$|\uparrow\rangle = \cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} |1\rangle, \quad |\downarrow\rangle = -\sin \frac{\theta}{2} |0\rangle + \cos \frac{\theta}{2} |1\rangle, \quad (3)$$

where  $\tan \theta = \Delta/\epsilon$ , Hamiltonian (2) takes the form of

$$H_c = \omega_r a^\dagger a + \sum_{j=1}^N \left[ \frac{\omega_a}{2} \sigma_j^z - g_j (a + a^\dagger) (1 - 2\bar{n} - \cos \theta \sigma_j^z + \sin \theta \sigma_j^x) \right], \quad (4)$$

where  $\omega_a = \sqrt{\epsilon^2 + \Delta^2}$  is the qubit splitting,  $\sigma^x$  and  $\sigma^z$  are the Pauli matrices in the  $\{|\uparrow\rangle, |\downarrow\rangle\}$  basis. The qubits are set to work at their optimal points ( $\bar{n} = 1/2$ ,  $\omega_a = \Delta$  and  $\theta = \pi/2$ , corresponding to  $\tilde{\sigma}_x$  basis), where they are immune to the charge noise and possess long decoherence time. Then, the Hamiltonian (4) reduces to

$$H_c = \omega_r a^\dagger a + \sum_{j=1}^N \left[ \frac{\Delta}{2} \sigma_j^z - g_j (a + a^\dagger) \sigma_j^x \right]. \quad (5)$$

Neglecting fast oscillating terms using the rotating-wave approximation lead Hamiltonian (5) to the usual Jaynes-Cummings form

$$H_{JC} = \omega_r a^\dagger a + \sum_{j=1}^N \left[ \frac{\Delta_j}{2} \sigma_j^z - g_j (a^\dagger \sigma_j^- + a \sigma_j^+) \right]. \quad (6)$$

Meanwhile, driving in the form of

$$h = \varepsilon(t) a^\dagger e^{-i\omega_d t} + \varepsilon^*(t) a e^{i\omega_d t} \quad (7)$$

on the resonator can be obtained [2] by capacitively coupling it to a microwave source with frequency  $\omega_d$  and amplitude  $\varepsilon(t)$ . Depending on the frequency, phase, and amplitude of the drive, different logical operations for qubit can be realized. To get fast gate, we work with large amplitude driving fields, where quantum fluctuations are very small compare with the drive amplitude, and thus the drive can be considered as a classical field. In this case, it is convenient to displace the field operators using the time-dependent displacement operator [13]:

$$D(\alpha) = \exp \left( \alpha a^\dagger - \alpha^* a \right). \quad (8)$$

Under this transformation, the field  $a$  goes to  $a + \alpha$  where  $\alpha$  is a c-number representing the classical part of the field. Choosing

$$i\dot{\alpha} = \omega_r \alpha + \varepsilon(t) \exp(-i\omega_d t) \quad (9)$$

to eliminate the direct drive on the resonator, then the displaced Hamiltonian reads [2]

$$H_D = \omega_r a^\dagger a + \sum_{j=1}^N \left\{ \frac{\Delta_j}{2} \sigma_j^z - g_j \left[ (a + \alpha) \sigma_j^+ + \text{H.c.} \right] \right\}. \quad (10)$$

Under resonant driving ( $\Delta_j = \omega_d$ ), the drive amplitude is independent of time, and change to a frame rotating at  $\omega_d$ , the displaced Hamiltonian reads

$$H_{\text{RF}} = \delta a^\dagger a + \sum_{j=1}^N \left[ \frac{\Omega}{2} \sigma_x^j - g_j (a \sigma_j^+ + a^\dagger \sigma_j^-) \right]. \quad (11)$$

where  $\delta = \omega_r - \omega_d$  and  $\Omega = 2g\varepsilon/\delta$  is the Rabi frequency.

Rotate to the eigen basis of  $\sigma_x$

$$\begin{aligned} H_{RF} &= H_0 + H_{int}, \\ H_0 &= \delta a^\dagger a + \frac{\Omega}{2} \sum_{j=1}^N \sigma_z^j, \\ H_{int} &= -\frac{1}{2} \sum_{j=1}^N [g_j a (\sigma_z^j + |+\rangle_j \langle -| - |-\rangle_j \langle +|) + \text{H.c.}] . \end{aligned} \quad (12)$$

In the interaction picture with respect to  $H_0$  the interaction Hamiltonian reads [12]

$$H_1 = -\frac{1}{2} \sum_{j=1}^N g_j a e^{-i\delta t} (\sigma_z^j + e^{i\Omega t} |+\rangle \langle -| - e^{-i\Omega t} |-\rangle \langle +|) + \text{H.c.} \quad (13)$$

In the case of  $\Omega \gg \{\delta, g\}$ , we can omitting the fast oscillation terms (of frequencies  $\Omega \pm \delta$ ) and only keep the oscillation frequency of  $\delta$ , then the Hamiltonian reads

$$H_2 = -\frac{1}{2} (a e^{-i\delta t} + a^\dagger e^{i\delta t}) \sum_{j=1}^N g_j \sigma_j^z. \quad (14)$$

Rotate back to the eigen basis of  $\sigma_z$ , the Hamiltonian reads

$$H_3 = -\frac{1}{2} (a e^{-i\delta t} + a^\dagger e^{i\delta t}) \sum_{j=1}^N g_j \sigma_j^x. \quad (15)$$

For  $N = 2$ , the corresponding closed Lie algebra for Hamiltonian (15) is  $\{1, a \sigma_j^x, a^\dagger \sigma_j^x, \sigma_1^x \sigma_2^x\}$ . The time evolution of Hamiltonian (15) is the product of their exponentials. Clearly, the first term represents a global phase factor, and thus can be neglected. The middle terms involve real excitation of the cavity field state, and thus entangle the qubits with the cavity field. The last term denotes a two qubits operation without entanglement with the cavity field. The time evolution operator can be written as [6]

$$U_2 = \exp(-2iA_2 \sigma_1^x \sigma_2^x) \left[ \prod_{j=1}^2 \exp(-iB_2^j a \sigma_j^x) \right] \left[ \prod_{j=1}^2 \exp(-i(B_2^j)^* a^\dagger \sigma_j^x) \right] \quad (16)$$

with

$$B_2^j = \frac{g_j}{i\delta} (e^{-i\delta t} - 1), \quad A_2 = \frac{g_1 g_2}{\delta} \left[ \frac{1}{i\delta} (e^{i\delta t} - 1) - t \right]. \quad (17)$$

It is clear that the whole time evolution operator is not a periodical function but  $B_2^j$  is and it vanishes at intervals

$$\delta T_n = 2n\pi \quad (18)$$

with  $n$  being a positive integer. At these time intervals, the time evolution operator reduce to

$$U_2(T_n) = \exp [-2iA_2(T_n)\sigma_1^x\sigma_2^x] \quad (19)$$

with  $A_2(T_n) = -g_1g_2T_n/\delta$ .

The reduced operator is equivalent to a two qubits system with Hamiltonian of the type of  $\sim \sigma_1^x\sigma_2^x$ . This two qubits operation is achieved without the entanglement with the cavity field state, and thus the cavity field do not transfer population to the qubits system. Therefore, the operation is insensitive to the cavity field state, the equilibrium state of which is usually a mixed state at finite temperature. The operation also remove the constrain of large detuning ( $\delta \gg g$ ), and  $T_1 \sim 1/g$  for  $\delta \sim g$ , which is comparable to the resonant coupling strategy. This shows fast gate operation can be achieved. Geometrically, the cavity state traverses cyclically and returns to its original phase space coordinates at intervals  $T_n$  leaving an geometric phase equals to the area of the trajectory [6]. This is shown to be a unconventional geometric phase factor [14], which still depends only on global geometric features and is robust against random operation errors [15], and thus high-fidelity two-qubit operation can be achieved [7].

This gate operation can be readily scale up to multiqubits scenario. For the purpose of simplicity, we assume  $g_j = g$  and define the collective spin operators as  $J_\nu = \sum_{j=1}^N \sigma_j^\nu$  with  $\nu = x, y, z$ . In this case, the time evolution operator can be written as

$$U_N = \exp (-iA_N J_x^2) \exp (-iB_N a J_x) \exp \left( -iB_N^* a^\dagger J_x \right) \quad (20)$$

with

$$B_N = \frac{g}{i\delta} \left( e^{-i\delta t} - 1 \right), \quad A_N = \frac{g^2}{\delta} \left[ \frac{1}{i\delta} \left( e^{i\delta t} - 1 \right) - t \right]. \quad (21)$$

Similarly, cavity field state insensitive operator

$$U_N(T_n) = \exp [-iA_N(T_n)J_x^2] \quad (22)$$

can be obtained at time intervals  $\delta T_n = 2n\pi$ . It is obvious that  $A_N \sim A_2$ , i.e., the time needed for this gate operation is comparable to that of the two-qubit case. This is another distinct merit of our proposed gate operation: the gate speed is not slowed down with the increasing involved qubits. Therefore, this merit enables efficient construction of entanglement and error correction code [10].

To sum up, scheme for coupling superconducting charge qubits via a cavity is proposed. The qubits are working at their optimal points, the time evolution of the interaction is shown to be insensitive to the initial state of the cavity field. This scheme enables fast gate operation and is readily scalable to multiqubit scenario.

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